Learning Objectives
Analisis Varians:

pengamatan lebih dari 2 sampel/macam/ perlakuan
efek perlakuan thd variabel diamati

1. Describe Analysis of Variance (ANOVA)
2. Compare Experimental Designs
3. Test the Equality of 2 or More Means
   - Completely Randomized Design (CRD=RAL)
   - Randomized Block Design
   - Factorial Design

Experimental Design

Rancangan Percobaan

Analysis of Variance

(Kwadrat SD= S^2) = $\sqrt{\frac{\sum(X-\overline{X})^2}{(n-1)}}$

A analysis of variance is a technique that partitions the total sum of squares of deviations of the observations about their mean into portions associated with independent variables in the experiment and a portion associated with error

Perbedaan dasar:

Analisis Varians: Efek Pada Percobaan lebih dari dua sampel/klmpk perlakuan (RA. Fisher, 1918)

Uji-T test: efek dua perlakuan

Completely Randomized Design (CRD=RAL)

1. Experimental Units (Subjects) Are Assigned Randomly to Treatments
   - Subjects are Assumed Homogeneous
2. One Factor or Independent Variable
   - 2 or More Treatment Levels or Classifications
3. Analyzed by One-Way ANOVA

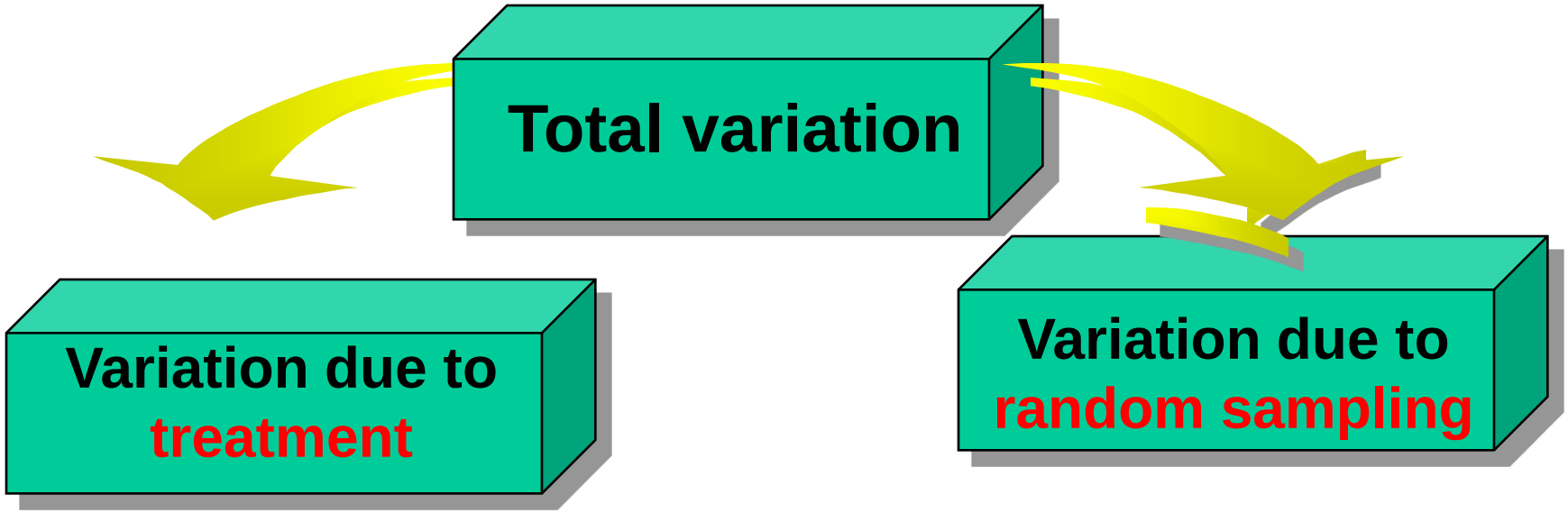
Catatan : RAL-Pola searah : percobaan 1 faktor perlakuan t.a. bbrp mcm level/dosis dg unit percob homogen dan variabel pengganggu diabaikan/dikemdalikan

Review: Completely Randomized Design (one-factor design)

- Experimental units are *relatively homogeneous*.
- Experiment will use very *few replicates*.
- Treatments are assigned to experimental units at *random*.
- Each treatment replicated the same number of times (*balance*). Un balaced, ok (contoh di praktikum)
- No accommodation made for *disturbing variables* (other sources of variation).
- High probability that a large fraction of the experimental units set out at the beginning of the study may be *lost or unavailable for measurement* at the appropriate time.

One-Way ANOVA : percob satu perlakuan ta. Bbrp level/unit homogen

Partitions Total Variation



- Sum of Squares Among
- Sum of Squares Between
- **Sum of Squares Treatment (SST)**
- Among Groups Variation
- Sumber diketahui

- Sum of Squares Within
- **Sum of Squares Error (SSE)**
- Within Groups Variation
- Sumber tidak diketahui

RAL (Rancangan Acak Lengkap)

Penerapan perlakuan terhadap unit percobaan dilakukan secara acak terhadap seluruh unit percobaan. Contoh, suatu percobaan melibatkan enam buah perlakuan (P1, P2, P3, P4, P5, P6) dan setiap perlakuan diulang sebanyak tiga kali. Dengan demikian unit percobaan yang dilibatkan sebanyak 3x6 = 18 unit percobaan. Pengacakan perlakuan dilakukan langsung terhadap 18 unit percobaan. Sehingga bagan percobaannya dapat digambarkan sebagai berikut:

P1	**P2**	**P1**	**P3**	**P5**	**P1**
P6	**P4**	**P3**	**P4**	**P5**	**P2**
P6	**P6**	**P4**	**P5**	**P2**	**P3**

POSISI TERNAK DALAM KANDANG
3 perlakuan x 3 ulangan

A	B	B
C	A	C
B	C	A

One-Way ANOVA Summary Table

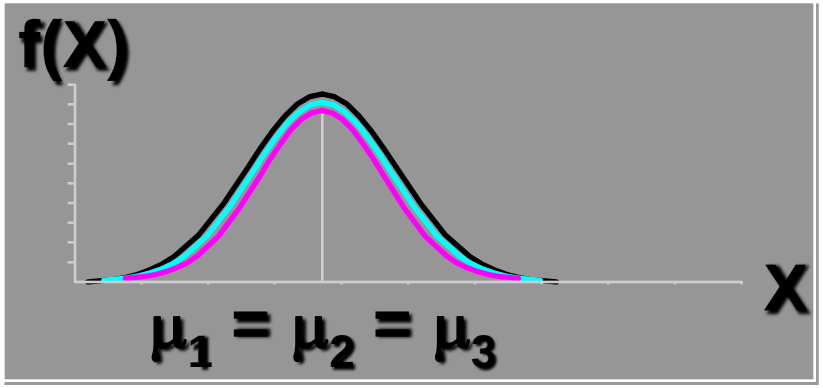
Source of Variation	Degrees of Freedom	Sum of Squares	Mean Square (Variance)	F
Treatment	t - 1	SST	MST = SST/(t - 1)	MST/ MSE
Error	tr - t	SSE	MSE = SSE/(tr - t)	
Total	tr - 1	SS(Total) = SST+SSE		

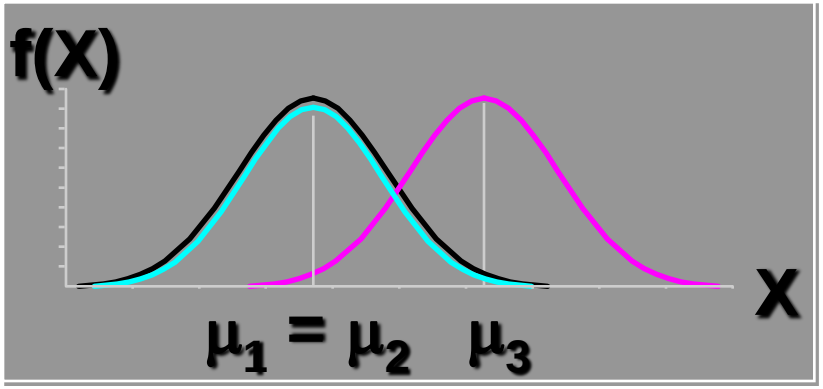
One-Way ANOVA

1. Compares 2 Types of Variation to Test Equality of Means
2. Comparison Basis Is Ratio of Variances
3. If Treatment Variation Is Significantly Greater Than Random Variation then Means Are **Not** Equal
4. Variation Measures Are Obtained by 'Partitioning' Total Variation

One-Way ANOVA F-Test Hypotheses

- H_0: $\mu_1 = \mu_2 = \mu_3 = \ldots = \mu_t$
  - All Population Means are Equal
  - No Treatment Effect
- H_a: Not All μ_i Are Equal
  - At Least 1 Pop. Mean is Different
  - Treatment Effect
  - NOT $\mu_1 \neq \mu_2 \neq \ldots \neq \mu_t$





Tabel Sidik Ragam

Sumber keragaman	Derajat bebas (DB)	Jumlah kuadrat (JK)	Kuadrat tengah (KT)	F-hitung
Ulangan sama $r_1=r_2= \ldots = r_t =r$				
Perlakuan	t-1	JKP	KTP	KTP/KTG
Galat	t(r-1)	JKG	KTG	
Total	tr-1	JKT		
Ulangan tidak sama $r_1 \neq r_2 \neq \ldots \neq r_t$				
Perlakuan	t-1	JKP	KTP	KTP/KTG
Galat	$\sum(r_i-1)$	JKG	KTG	
Total	$\sum r_i-1$	JKT		

Completely Randomized Design

•Experimental Design - Completely randomized design (CRD)
•Sampling Design - One-way classification design

Assumptions:

- Independent random samples (results of one sample do not effect other samples).
- Samples from normal population(s).
- Mean and variance for population i are respectively, μ_i and σ^2.

Model: $y_{ij} = \mu + \alpha_i + \varepsilon_{ij}$ $\quad E(y_{ij}) = \mu + \alpha_i$

AOV model

overall mean

effect due to population i

random error ~ $N(0,\sigma^2)$

Requirement for μ to be the overall mean:

$$\sum_{i=1}^{t} \alpha_i = 0$$

$H_0: \quad \alpha_1 = \alpha_2 = \cdots = \alpha_t = 0$

$H_a: \quad$ At least one of the α differ from 0

The Linier Model

$$y_{ij} = \mu + \tau_i + \varepsilon_{ij}$$

i = 1,2,..., t j = 1,2,..., r

y_{ij} = the observation in i^{th} treatment and the j^{th} replication

μ = overall mean

τ_i = the effect of the i^{th} treatment

ε_{ij} = random error

Contoh :

1. Uji coba produk .

2. Berat lahir sapi PO di lokasi beda

Home Products, Inc. is considering marketing a long-lasting car wax. Three different waxes (Type 1, Type 2, and Type 3

In order to test the durability of these waxes, 5 new cars were waxed with Type 1, 5 with Type 2, and 5 with Type 3. Each car was then repeatedly run through an automatic carwash until the wax coating showed signs of deterioration.

Pertanyaan :

Home Products, Inc. must decide which wax to market. Are the three waxes equally effective?

- Hypotheses

H_0: $\mu_1 = \mu_2 = \mu_3$

H_a: Not all the means are equal, where:

μ_1 = mean number of washes for Type 1 wax

μ_2 = mean number of washes for Type 2 wax

μ_3 = mean number of washes for Type 3 wax

Observation	Wax Type 1	Wax Type 2	Wax Type 3
1	27	33	29
2	30	28	28
3	29	31	30
4	28	30	32
5	31	30	31
Sample Mean	29.0	30.4	30.0
Sample Variance	2.5	3.3	2.5

- Mean Square Between Treatments, Since the sample sizes are all equal:

  $\mu = (x_1 + x_2 + x_3)/3 = (29 + 30.4 + 30)/3 = 29.8$

  $SST = 5(29–29.8)^2 + 5(30.4–29.8)^2 + 5(30–29.8)^2 = 5.2$

  $MST = 5.2/(3 - 1) = 2.6$

- Mean Square Error

  $SSE = 4(2.5) + 4(3.3) + 4(2.5) = 33.2$

  $MSE = 33.2/(15 - 3) = 2.77$

- ANOVA Table

Source of Variation	Sum of Squares	Degrees of Freedom	Mean Squares	*F*
Treatments	5.2	2	2.60	.9398
Error	33.2	12	2.77	
Total	38.4	14		

- Test Statistic

  F = MST/MSE = 2.6/2.77 = .939

- Conclusion

  Since $F = .939 < F_{.05} = 3.89$, we cannot reject H_0. There is insufficient evidence to conclude that the mean number of washes for the three wax types are not all the same.

- Rejection Rule

  Using test statistic: Reject H_0 if $F > 3.89$

  Using p-value: Reject H_0 if p-value $< .05$

  where $F_{.05} = 3.89$ is based on an F distribution with 2 numerator degrees of freedom and 12 denominator degrees of freedom

2**. Contoh bidang peternakan: (Home work), latihan mandiri)**

Percobaan dilakukan untuk mengetahui apakah terdapat perbedaan
rata rata bobot lahir (BL, kg) sapi llokal PO
Yang dilahirkan di 4 lokasi (L1,L2,L3,L4) yang berbeda

Data BL adalah sebagai berikut:

L1	L2	L3	L4
28	31	22	25
30	30	19	30
32	32	20	35
25	29	18	28
30	32	22	29
24	33	24	26

Contoh RAL

Ulangan	Perlakuan				
	P0	P1	P2	P3	P4
U1	14,7	16	15,2	13,3	13,1
U2	15,3	14,1	12,9	14,6	11,6
U3	14,3	15,5	13,5	13,4	13,1
U4	12,7	15,1	16,7	12,5	11,6
Total	57	60,7	58,3	53,8	49,4
Rata-rata	14,25	15,175	14,575	13,45	12,35
Sd	1,11205	0,8057	1,7192	0,86602	0,86602

a. Faktor Koreksi (FK)

$$FK = \frac{(\sum_{ij} Y_{ij})^2}{t \times r}$$

$$= \frac{(279,2)^2}{20} = 3897,63$$

b. Jumlah Kuadrat Total

$$JKtotal = (\sum_{ij} Y^2_{ij}) - FK$$

$$= (14,7^2 + 16^2 + \ldots\ldots\ldots + 11,6^2) - 3897,63$$

$$= 3935,82 - 3897,63$$

$$= 38,188$$

c. Jumlah Kuadrat Perlakuan

$$JK\ Perlakuan = \frac{\sum Y_i^2 + \ldots\ldots + Y_j^2}{Ulangan} - FK$$

$$= 57^2 + 60,7^2 + \ldots\ldots + 49,4^2 - 3897,63$$

$$= 3916,8 - 3897,63$$

$$= 19,163$$

d. Jumlah Kuadrat Galat

$$JK\ galat = JK\ total - JK\ perlakuan$$

$$= 38,188 - 19,163$$

$$= 19,025$$

e. db Perlakuan

$$db\ Perlakuan = t - 1$$

$$= 5 - 1$$

$$= 4$$

f. db Galat

$$db\ Galat = t\,(r-1)$$

$$= 5\,(4-1)$$

$$= 15$$

g. Kuadrat Tengah Perlakuan (KT perlakuan)

$$KTperlakuan = \frac{JK\ perlakuan}{db\ perlakuan}$$

$$= \frac{19,163}{4} = 4,79075$$

h. Kuadrat Tengah Galat (KT galat)

$$KTgalat = \frac{JK\ galat}{db\ galat}$$

$$= \frac{19,025}{15} = 1,26833$$

i. F hitung

$$F\ hitung = \frac{KT\ perlakuan}{KT\ galat}$$

$$= \frac{4,79075}{1,26833} = 3,7772$$

Tabel ANOVA

SK	db	JK	KT	F hitung	F tabel	
					5%	1%
Perlakuan	4	19,163	4,79075	3,7772*	3,06	4,89
Galat	15	19,025	1,26833			
Total	19	38,188				

Kesimpulan : *F hitung > F table 5%, maka menunjukkan bahwa perlakuan memberikan pengaruh yang berbeda nyata (P<0,05).

Uji Lanjut Jarak Berganda Duncan

	2	3	4	5	6	7
JND	3,01	3,16	3,25	3,31	3,36	3,38
JNT	1,6945	1,7794	1,83014	1,8639	1,8921	1,9033

$$JNT = \sqrt{\frac{KTg}{u}} \times \text{tabel duncan}$$

$$= \sqrt{\frac{1,26833}{4}} \times \text{table Duncan}$$

Perlakuan	Rata - rata	Notasi
P4	12,35	a
P3	13,45	ab
P0	14,25	b
P2	14,575	b
P1	15,175	b

b.) Pengujian Rataan Perlakuan :

1.) DNT (Beda Nyata Terkecil)

$SE = \sqrt{2 \text{ KT Galat/r}} = \sqrt{2 \times 0,00037/3} = \sqrt{0,0002}$

$= 0,0141$

DNT 1% = t(1% db galat 8) x SE = 3,36 x 0,0141

= 0,0473

Perlakuan (Kepadatan)	Rataan
3	1,29
4	1,3766
5	1,4666
6	1,41
7	1,3633

Diurutkan :

perlakuan	Rataan	Notasi
3	1,29	a
7	1,3633	b
4	1,3766	b
6	1,41	b
5	1,4666	c

Perlakuan kandang dengan kepadatan 5 ekor memberikan bobot badan paling tinggi pada ayam pedaging.

BNJ (Beda Nyata Jujur)

$SE = \sqrt{KT \text{ Galat/r}}$

$= \sqrt{0,00037/3} = \sqrt{0,0001} = 0,01$

BNJ 1% = qp. db galat - α(1% x SE

= 5,68 x 0,01 = 0,0568

Perlakuan	rataan	Notasi
3	1,29	a
7	1,3633	b
4	1,3766	b
6	1,41	bc
5	1,4666	c

Perlakuan kandang dengan kepadatan 5 ekor memberikan bobot badan yg paling tinggi pada ayam pedaging tetapi pengaruhnya tdk berbeda dgn perlakuan kandang dgn kepadatan 6 ekor ayam pedaging.

2) Duncan.

JNT α% (α% db galat · pg x SE

$SE = \sqrt{KT \text{ Galat/r}} = \sqrt{0,00037/3} = \sqrt{0,0001} = 0,01$

JND 1% :	4,79	5,00	5,19	5,23
JNT 1% :	0,0479	0,05	0,0519	0,0523
	b	①	②	③

Perlakuan	Rataan	Notasi
3	1,29	a
7	1,3633	b
4	1,3766	b
6	1,41	b
5	1,4666	c

CONTOH:

Percobaan untuk mencari metode promosi yang paling efektif → memberikan hasil penjualan yang tertinggi

Metode: A (brosur), B (spanduk), dan
C (dari pintu ke pintu)

Ulangan: 5 kali

Satu metode diterapkan pada satu lokasi tertentu (tipe lokasi & masyarakatnya relatif sama), kemudian diamati hasil penjualannya selama periode waktu tertentu.

Model

$$Y_{ij} = \mu + \tau_i + \varepsilon_{ij} \;\; ; \;\; i = 1,2,..., p \; ; \; j = 1,2,..., r.$$

dengan :

Y_{ij} = respons pada perlakuan ke-i, ulangan ke-j

μ = rataan umum

τ_i = pengaruh metode ke – i

ε_{ij} = error atau galat pada perlakuan ke-i, ulangan ke-j

Rancangan Acak Lengkap

Pengacakan? Syarat ??

Data yang terkumpul sebagai berikut:

Metode	Hasil penjualan lokasi ke					Rataan	Jumlah
	1	2	3	4	5		
A	18	20	15	18	21	18.4	92
B	10	16	12	13	15	13.2	66
C	21	25	18	23	20	21.4	107
						17.7	265

Berapa nilai-nilai dugaan parameter model $(\hat{\mu}, \hat{\tau}_i)$?

Penguraian jumlah kuadrat ➔ JKT = JKP + JKG

Bagaimana menguji keefektifan ketiga metode tersebut ?

➔ Tabel Anova, Hipotesis , Statistik Uji?

Penguraian Jumlah Kuadrat

$$\hat{\mu} = \bar{Y}_{..} \; ; \quad \hat{\mu}_{\mathrm{i}} = \bar{Y}_{i.} \; ; \; \varepsilon_{\mathrm{ij}} = e_{ij} = Y_{ij} - \hat{Y}_{ij} = Y_{ij} - \bar{Y}_{i.}$$

$$Y_{ij} - \bar{Y}_{..} = Y_{ij} - \bar{Y}_{i.} + \bar{Y}_{i.} - \bar{Y}_{..}$$

$$(Y_{ij} - \bar{Y}_{..}) = (Y_{ij} - \bar{Y}_{i.}) + (\bar{Y}_{i.} - \bar{Y}_{..})$$

Jika kedua ruas dikuadratkan:

$$(Y_{ij} - \bar{Y}_{..})^2 = (Y_{ij} - \bar{Y}_{i.})^2 + (\bar{Y}_{i.} - \bar{Y}_{..})^2 + 2(Y_{ij} - \bar{Y}_{i.})(\bar{Y}_{i.} - \bar{Y}_{..})$$

$$\sum_i \sum_j (Y_{ij} - \bar{Y}_{..})^2 = \sum_i \sum_j (Y_{ij} - \bar{Y}_{i.})^2 + \sum_i \sum_j (\bar{Y}_{i.} - \bar{Y}_{..})^2$$

$$\mathrm{JKT} = \mathrm{JKP} + \mathrm{JKG}$$

Analisis of Variance (Anova)

Sumber	db	JK	KT	F	$F_{0.05(2,12)}$
Metode	2	172.13	86.07	14.11	3.89
Error	12	73.2	6.1		
Total	14	245.33			

Asumsi:
Kenormalan
Kehomogenan ragam
Kebebasan galat

H_0: $\tau_1 = \tau_2 = \tau_3 = 0$

H_1: Paling sedikit ada satu $\tau_i \neq 0$

Karena Fhit > Ftab → Tolak H0

→ ada perbedaan pengaruh perlakuan

(antar metode memberikan hasil penjualan yang berbeda)

Mana yang berbeda? A dg B, B dg C, A dg C, ataukah A,B,C berbeda ???